FOCM 2014 - Workshop C5
Special Functions and Orthogonal Polynomials

C5 - December 18, 14:30 – 15:00

Extending Askey tableau by the inclusion of Krall and exceptional polynomials

Antonio J. Durán
Universidad de Sevilla, Spain
duran@us.es

Krall and exceptional polynomials are two of the more important extensions of the classical families of Hermite, Laguerre and Jacobi. On the one hand, Krall or bispectral polynomials are orthogonal polynomials which are also eigenfunctions of a differential operator of order bigger than two (and polynomial coefficients). The first examples were introduced by H. Krall in 1940, and since the eighties a lot of effort has been devoted to this issue (with contributions by L.L. Littlejohn, A.M. Krall, J. and R. Koekoek. A. Grünbaum and L. Haine (and collaborators), K.H. Kwon (and collaborators), A. Zhedanov, P. Iliev, and many others). On the other hand, exceptional polynomials are orthogonal polynomials which are also eigenfunctions of a second order differential operator, but they differ from the classical polynomials in that their degree sequence contains a finite number of gaps, and hence the differential operator can have rational coefficients. In mathematical physics, these functions allow to write exact solutions to rational extensions of classical quantum potentials. Exceptional polynomials appeared some seven years ago, but there has been a remarkable activity around them mainly by theoretical physicists (with contributions by D. Gómez-Ullate, N. Kamran and R. Milson, Y. Grandati, C. Quesne, S. Odake and R. Sasaki, and many others). Taking into account these definitions, it is scarcely surprising that no connection has been found between Krall and exceptional polynomials. However, if one considers difference operators instead of differential ones (that is, the discrete level of Askey tableau), something very exciting happens: Duality (i.e., swapping the variable with the index) interchanges Krall discrete and exceptional discrete polynomials. This unexpected connection of Krall discrete and exceptional polynomials allows a nice and important extension of Askey tableu. Also, this worthy fact can be used to solve some of the most interesting questions concerning exceptional polynomials; for instance, to find necessary and sufficient conditions such that the associated second order differential operators do not have any singularity in their domain.

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Orthogonal and para-orthogonal polynomials on the unit circle

A. Sri Ranga
UNESP - Universidade Etadual Paulista, Brazil
ranga@ibilce.unesp.br

When a nontrivial measure μ on the unit circle satisfies the symmetry $d\mu(e^{i(2\pi-\theta)}) = -d\mu(e^{i\theta})$ then the associated orthogonal polynomials on the unit circle, say S_n, are all real. In this case, in [3], Delsarte and Genin have shown that the two sequences of para-orthogonal polynomials $\{zS_n(z) + S_n^*(z)\}$ and $\{zS_n(z) - S_n^*(z)\}$ satisfy three term recurrence formulas and have explored some further consequences of these sequences of polynomials such as their connections to sequences of orthogonal polynomials on the interval $[-1, 1]$. Even though results presented in Delsare and Genin [4] extend these partly to include any nontrivial measures on the unit circle, only recently, in [2] (and also [1]), the extension associated with the para-orthogonals polynomials $zS_n(z) - S_n^*(z)$ was studied extensively. The results given in [2], especially from the point of view of three term recurrence, provide also as a nice application a characterization for any pure points in the measure. The main objective of the present contribution is to provide some recent developments concerning the extension for the para-orthogonals polynomials $zS_n(z) + S_n^*(z)$ to cover all nontrivial measures on the unit circle.

References

[1] K. Castillo, M. S. Costa, A. Sri Ranga and D. O. Veronese, A Favard type theorem for orthogonal polynomials on the unit circle from a three term recurrence formula, J. Approx. Theory, 184 (2014), 146-162.

[2] M.S. Costa, H.M. felix and A. Sri Ranga, Orthogonal polynomials on the unit circle and chain sequences, J. Approx. Theory, 173 (2013), 14-32.

[3] P. Delsarte and Y. Genin, The split Levinson algorithm, IEEE Trans. Acoust. Speech Signal Process, 34 (1986), 470-478.

[4] P. Delsarte and Y. Genin, The tridiagonal approach to Szegő's orthogonal polynomials, Toeplitz linear system, and related interpolation problems, SIAM J. Math. Anal., 19 (1988), 718-735.

Joint work with Cleonice F. Bracciali (Universidade Estadual Paulista, Brazil) and A. Swaminathan (IIT Roorkee, India).

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On orthogonal polynomials associated with perturbations of Hankel matrices

Luis E. Garza
Universidad de Colima, México
garzaleg@gmail.com

In this contribution, we study algebraic properties of orthogonal polynomials associated with perturbations of Hankel matrices. In particular, we give explicit expressions for such polynomials, and obtain some properties associated with their zeros.

Joint work with K. Castillo (Universidad Estadual Paulista, Brazil), D. Dimitrov (Universidad Estadual Paulista, Brazil) and F. Rafaeli (Universidad Estadual Paulista, Brazil).

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Kissing polynomials

Arieh Iserles
University of Cambridge, United Kingdom
ai@damtp.cam.ac.uk

Our point of departure are Gauss-like methods for highly oscillatory quadrature of integrals $\int_{-1}^{1} f(x)e^{\mathrm{i}\omega x}\mathrm{d}x$, $\omega \gg 1$, employing orthogonal polynomials with a complex-valued weight function. Such polynomials pose fascinating questions of existence and their zeros, plotted in the complex plane as a function of the underlying frequency ω, exhibit an intriguing pattern of "kissing". All this will be analysed and elucidated in the talk. Our main tool is asymptotic analysis for $\omega \gg 1$ of Hankel determinants, represented by multivariate highly oscillatory integrals, and their generalisations.

Joint work with Andreas Asheim (NTNU Trondheim), Alfredo Deaño (KU Leuven) and Daan Huybrechs (KU Leuven).

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Orthogonal polynomials and integral transforms

Ana F. Loureiro
University of Kent, U.K.
A.Loureiro@kent.ac.uk

In this talk I will explain how operating with certain integral transforms over polynomial sequences is a useful tool to obtain and deduce properties of one sequence based on the other. A special attention will be given to certain d-orthogonal polynomial sequences, which basically are polynomial sequences satisfying a recurrence relation of order $d+1$. When $d=1$, we recover the orthogonal case. Examples of some known polynomial sequences with a plank of applications will be used to illustrate the usefulness of the technique. Among the targeted sequences, some semiclassical polynomials will arise.

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Explicit formulas for OPUC and para-orthogonal polynomials for measures which are modifications of Lebesque measure

Cleonice Bracciali
UNESP - Universidade Estadual Paulista, Brazil
cleonice@ibilce.unesp.br

We consider nontrivial probability measures, obtained as simple modifications of the Lebesgue measure, which include mass points at $z=1$ and $z=i$. The orthogonal polynomials on the unit circle (OPUC), the para-orthogonal polynomials and Toeplitz matrices associated with these measures are presented, through explicit formulas for the Verblunsky coefficients.

Joint work with A. Sri Ranga (UNESP - Universidade Estadual Paulista, Brazil).

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Spectral Orthogonal Polynomals and Differential Galois Theory

Primitivo Acosta-Humánez
Universidad del Atlántico, Colombia
primi@intelectual.co

In this talk we present a Differential Galois Theory approach to study the Schrödinger equation with polynomial potentials. We obtain the algebraic spectrum, values of energy in which the Schrodinger equation is integrable in Galoisian sense, of harmonic and anharmonic oscillators. In particular, for the sextic anharmonic oscillator we recover the classical Bender - Dunne polynomials as spectral orthogonal polynomials, that is, the algebraic spectrum of the Schrödinger equation with polynomial potential of degree six which corresponds to the zeroes of the Bender-Dunne spectral polynomials. Generalizations of this approach, such as decatic potentials, are also presented.

Joint work with Henock Venegas (Universidad del Atlántico, Colombia).

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Semi-classical orthogonal polynomials and the Painlevé equations

Peter Clarkson
University of Kent, UK
P.A.Clarkson@kent.ac.za

In this talk I shall discuss the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation. I will show that for some semi-classical weights, the coefficients in the recurrence relation can be expressed in terms of Wronskians that arise in the description of special function solutions of a Painlevé equation. The orthogonal polynomials discussed will include semi-classical Laguerre and Hermite weights, orthogonal polynomials with discontinuous weights and semi-classical generalizations of the Charlier and Meixner polynomials, which are discrete orthogonal polynomials.

Joint work with Kerstin Jordaan (University of Pretoria, South Africa) and J G Smith (University of Kent, United Kingdom).

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Bispectrality, the Darboux process and time-band limiting

Mirta María Castro Smirnova
University of Seville, Spain
mirta@us.es

We consider a few examples illustrating connections among the notions in the title.

Joint work with Alberto Grünbaum (University of California, Berkeley).

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Orthogonal Rational Functions and non-stationary stochastic processes

Laurent Baratchart
INRIA Sophia-Antipolis, France
Laurent.Baratchart@inria.fr

We present a generalization of the Szegö theory of orthogonal polynomials on the unit circle to orthogonal rational functions. Just like the Kolmogorov-Krein-Szegö theorem may be interpreted as an asymptotic estimate of the prediction error for stationary stochastic processes, the present theory yields an asymptotic estimate of the prediction error for certain, possibly non-stationary, stochastic processes. The latter admit a spectral calculus where the time-shift corresponds to multiplication by elementary Blaschke products of degree 1 (that reduce to multiplication by the independent variable in the stationnary case). When the poles of the best predictor tend to a point on the unit circle where the spectral density is non-zero, the prediction error goes to zero, that is, the process is asymptotically deterministic.

Joint work with Leonid Golinskii (Mathematics Division, Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine) and Stanislas Kupin (Mathematics Department, University Bordeaux I, France).

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Multiple orthogonal polynomials associated with an exponential cubic weight

Walter Van Assche
KU Leuven, Belgium
walter@wis.kuleuven.be

We consider multiple orthogonal polynomials associated with the exponential cubic weight e^{-x^3} over two contours in the complex plane. We give the basic properties of these polynomials, including the Rodrigues formula and nearest-neighbor recurrence relations. It turns out that the recurrence coefficients are related to a discrete Painlevé equation. The asymptotics of the recurrence coefficients, the ratio of the diagonal multiple orthogonal polynomials and the (scaled) zeros of these polynomials are also investigated.

Joint work with Galina Flipuk (Univerity of Warsaw, Poland) and Lun Zhang (Fudan University, Shanghai, China).

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Quadratic Algebras of Orthogonal Polynomials

Sarah Post
University of Hawaii, USA
spost@hawaii.edu

In this talk, we will discuss the connection between superintegrable systems and classical systems of orthogonal polynomials in particular in the expansion coefficients between separable coordinate systems, related to representations of the (quadratic) symmetry algebras. This connection allows us to extend the Askey scheme of classical orthogonal polynomials and the limiting processes within the scheme. In particular, for superintegrable systems in 3D, the polynomial representations of quadratic algebras are given in terms of two-variable polynomials and the two-variable analog of the Askey scheme, including the quadratic Racah algebra, will be discussed.

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G-function of Meijer and generalized hypergeometric function: interplay of new facts

Dmitrii Karp
Far Eastern Federal University, Russian Federation
dimkrp@gmail.com

We discuss various new properties of Meijer's G-function $G^{p,0}_{p,p}$, including integral and functional equations, nonnegativity conditions and number of zeros, convergence of measures with G-function density and regularization of integrals containing G-function. Some of these properties are then applied to derive new representations for generalized hypergeometric functions and establish some new and old facts about them. In particular, we prove log-convexity in upper parameters, demonstrate monotonicity of certain ratios and find new proofs for Luke's inequalities permitting their extension to wider parameter ranges. We further find an upper bound for the Gauss type generalized hypergeometric function not previously contained in the literature. Finally, using a different approach we give new two-sided bounds for the Bessel type generalized hypergeometric function of nonnegative argument.

The talk is based on joint work with J.L.Lopez and E.G.Prilepkina. The research has been supported by the Ministry of Education and Science of the Russian Federation under project 1398.2014.

C5 - December 20, 14:30 – 15:00

Applications of infinite matrices in the theories of orthogonal polynomials and operational calculus

Luis Verde-Star
Universidad Autonoma Metropolitana, Mexico
verde@xanum.uam.mx

We use some algebras of infinite matrices $[a_{j,k}]$, where the indexes run over all the integers, to study sequences of polynomials and formal power series and also for the construction of a general operational calculus that can be used to solve linear functional equations of several types.

We consider infinite matrices of the form $\sum_k D_k X^k$, where the D_k are diagonal matrices, X is a shift, and $D_k \neq 0$ for only a finite number of negative values of k. Several basic properties and characterizations of orthogonal polynomial sequences are expressed in terms of infinite matrices.

This work extends some of the results obtained in our previous papers

L. Verde-Star, Characterization and construction of classical orthogonal polynomials using a matrix approach, Linear Algebra Appl. 438 (2013) 3635–3648.

G. Bengochea, L. Verde-Star, Linear algebraic foundations of the operational calculi, Adv. Appl. Math. 47 (2011) 330–351.

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Branching formula for Macdonald-Koornwinder polynomials

Jan Felipe van Diejen
Universidad de Talca, Chile
diejen@inst-mat.utalca.cl

The six-parameter Macdonald-Koornwinder polynomials with hyperoctahedral symmetry form a multivariate generalization of the well-known Askey-Wilson polynomials. We present a branching formula that expands the Macdonald-Koornwinder polynomials in n+1 variables in terms of the n-variable polynomials. This formula allows one to construct the Macdonald-Koornwinder polynomials explicitly by induction in the number of variables.

Joint work with Erdal Emsiz (Pontificia Universidad Católica de Chile, Chile).

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Hermite-Padé Approximants for Angelesco Systems

Maxim Yattselev
Indiana University-Purdue University Indianapolis, USA
maxyatts@iupui.edu

I will discuss asymptotics of type II Hermite-Padé approximants (a vector of rational functions with a common denominator) for a vector of Cauchy transforms of analytic densities along any ray of multi-indices. It is assumed that the densities are supported on mutually disjoint intervals (an Angelesco system with complex weights). The formulae of strong asymptotics will be presented, the relevant Riemann surfaces discussed as well as some details of the local Riemann-Hilbert analysis.

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A q-generalization of the Bannai–Ito polynomials and the quantum superalgebra $\mathfrak{osp}_q(1|2)$

Luc Vinet
Université de Montréal, Canada
luc.vinet@umontreal.ca

A q-generalization of the Bannai–Ito polynomials is presented. These basic polynomials are obtained by considering the Racah problem for the quantum superalgebra $\mathfrak{osp}_q(1|2)$. A quantum deformation of the Bannai–Ito algebra is realized by the intermediate Casimir operators entering in the Racah problem. The relation between the q-analogs of the Bannai–Ito polynomials and the q-Racah/Askey-Wilson polynomials is discussed.

Joint work with Vincent X. Genest (CRM) and Alexei Zhedanov (Donetsk).

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Lifting q-difference operators in the Askey scheme of basic hypergeometric polynomials

Natig Atakishiyev
Instituto de Matemáticas, Unidad Cuernavaca, Universidad Nacional Autónoma de México, Mexico
natig@matcuer.unam.mx

We construct an explicit form of a q-difference operator that lifts the continuous q-Hermite polynomials $H_n(x|q)$ of Rogers into the Askey-Wilson polynomials $p_n(x;a,b,c,d|q)$ on the top level in the Askey q-scheme. This operator represents a special convolution-type product of four one-parameter q-difference operators of the form $\epsilon_q(c_q D_q)$, defined as Exton's q-exponential function $\epsilon_q(z)$ in terms of the Askey-Wilson divided q-difference operator D_q. We show also that one can determine another q-difference operator that transforms the orthogonality weight function for the continuous q-Hermite polynomials $H_n(x|q)$ of Rogers up to the weight function, associated with the Askey-Wilson polynomials $p_n(x;a,b,c,d|q)$.

Joint work with Mesuma Atakishiyeva (Facultad de Ciencias, Universidad Autónoma del Estado de Morelos, Cuernavaca, Morelos, México).

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Quasi-orthogonality of some ${}_pF_q$ hypergeometric polynomials

Kerstin Jordaan
University of Pretoria, South Africa
kjordaan@up.ac.za

We prove the quasi-orthogonality of some general classes of hypergeometric polynomials of the form

$$ {}_pF_q\left(\begin{matrix} -n, \beta_1+k, \alpha_3\ldots,\alpha_p \\ \beta_1,\ldots,\beta_q \end{matrix}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(\beta_1+k)_m(\alpha_3)_m\ldots(\alpha_p)_m}{(\beta_1)_m\ldots(\beta_q)_m}\frac{x^m}{m!} $$

for $k \in \{1,2,\ldots,n-1\}$ which do not appear in the Askey scheme for hypergeometric orthogonal polynomials. Our results include, as a special case, the order one quasi-orthogonal Sister Celine polynomials

$$ f_n(a,x) = {}_3F_2\left(\begin{matrix} -n, n+1, a \\ \frac{1}{2}, 1 \end{matrix}; x\right) = \sum_{m=0}^{n} \frac{(-n)_m(n+1)_m(a)_m}{\left(\frac{1}{2}\right)_m(1)_m}\frac{x^m}{m!} $$

with $a = 2$ and $a = 3/2$ considered by Dickenson in 1961. The location and interlacing of the real zeros of the quasi-orthogonal polynomials are also discussed.

Joint work with Sarah Jane Johnston (University of South Africa, South Africa).

C5 - Poster

The relationship between the fifth Painlevé equation and orthogonal polynomials

Peter Clarkson
University of Kent, United Kingdom
P.A.Clarkson@kent.ac.uk

Here we are concerned with orthogonal polynomials for a deformed Laguerre weight. It is shown that the coefficients of the three-term recurrence relation satisfied by the polynomials can be expressed in terms of Wronskians which involve Kummer functions. These Wronskians are related to special function solutions of the fifth Painlevé equation. Using this relationship we can explicitly write the recurrence relation coefficients in terms of exact solutions of the fifth Painlevé equation.

Joint work with J Smith (University of Kent, UK).

C5 - Poster

Bivariate Lagrange interpolation and quadrature formulas at the node sets of Lissajous curves

Wolfgang Erb
University of Lübeck, Germany
erb@math.uni-luebeck.de

Motivated by an application in Magnetic Particle Imaging, we study bivariate Lagrange interpolation at the node points of Lissajous curves. The resulting theory is a generalization of the polynomial interpolation theory developed for a node set known as Padua points. With appropriately defined polynomial spaces, we show that the node points of Lissajous curves allow unique interpolation. Further, these node sets can be used as sampling points for quadrature rules for integrals with product Chebyshev weights. An explicit formula for the Lagrange polynomials allows to compute the interpolating polynomial with a simple algorithmic scheme. Compared to the already established schemes of the Padua and Xu points, the numerical results for the proposed scheme show similar approximation errors and a similar growth of the Lebesgue constant.

Joint work with Christian Kaethner (University of Lübeck, Germany), Mandy Grüttner (University of Lübeck, Germany) and Thorsten M. Buzug (University of Lübeck, Germany).

C5 - Poster

Completed Stieltjes interlacing of zeros of different orthogonal polynomials

Alta Jooste
University of Pretoria, South Africa
alta.jooste@up.ac.za

Consider the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$. If the polynomials p_n and p_{n-m}, $m = 2, 3, \ldots, n-1$, have no common zeros, there exists a real polynomial of degree $m-1$, completely determined by the coefficients in the three term recurrence relation satisfied by the orthogonal sequence $\{p_n\}_{n=0}^{\infty}$, whose real simple zeros provide a set of points that, together with the zeros of p_{n-m}, completely interlace with the zeros of p_n, a property we refer to as completed Stieltjes interlacing. The conditions under which completed Stieltjes interlacing holds between the zeros of polynomials from different orthogonal sequences are studied and this leads to a set of points that can be applied as bounds for the extreme zeros of the polynomials. We apply our results to some discrete orthogonal polynomials and identify new bounds for the extreme zeros of these polynomials.

Joint work with Kerstin Jordaan (University of Pretoria, South Africa).

C5 - Poster

Application of Fourier series for modelling forest fires in the Russian Far East

Elena Oleinik
Far Eastern Federal University, Russia
oleinik.elena@gmail.com

In the report, we discuss the relationship between the frequency of forest fires in the Russian Far East and the bursts of solar activity. We demonstrate the cointegration between the two variables and estimate the parameters of the model involving a Fourier series. The model is then verified against the data and used to forecast the burned area. Such forecasting helps to reduce the environmental and economic damage.

Joint work with Elena Oleinik (Far Eastern Federal University, Russia).
